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*Publication date:*  
2001

*Document Version*  
Også kaldet Forlagets PDF

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*

Izadi-Zamanabadi, R. (2001). Structural Analysis Approach to Fault Diagnosis with Application to Fixed-wing Aircraft Motion.

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# Structural Analysis Approach to Fault Diagnosis with Application to Fixed-wing Aircraft Motion

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## Abstract

The paper presents a structural analysis based method for fault diagnosis purposes. The method uses the structural model of the system and utilizes the matching idea to extract system's inherent redundant information. The structural model is represented by a bipartite directed graph. FDI Possibilities are examined by further analysis of the obtained information. The method is illustrated by applying on the LTI model of motion of a fixed-wing aircraft.

## 1 Introduction

In complex systems with large number of sensors, continued operation of various subsystems has both economic and safety implications. The issue of obtaining information about various parameters and signals, which have to be monitored for fault detection purposes, becomes a rigorous task with the growing number of subsystems/actuators/sensors.

The structural approach ([5, 3, 7]), constitutes a general framework for providing information when the system becomes complex. The main objective of applying the structural approach is to identify the subsystems with inherent redundant information. This paper presents a novel approach to fault detection and isolation (FDI).

The paper presents the structural model for a system in form of a bipartite directed graph representation. The concept of matching is described and then a procedure for performing the matching is explained. Matching can be performed differently on a structural model. To identify the number of all possible matchings a procedure is introduced. The structural observability is considered and it is shown how it is related to fault detection and isolation issue. A linear model of a fixed-wing aircraft motion is then used to illustrate the applicability of the method. Finally, simulation result of an obtained residual is presented.

## 2 Structural model

Consider the system  $S$  as a set of components  $\bigcup_{i=1}^m C_i$ , each imposing one (or several) relations  $f_i$  between a set of variables  $z_j, j = 1, \dots, n$  i.e.

$$f_i(z_1, \dots, z_p) = 0, \quad 1 < p \leq n \quad (1)$$

$f_i$  can represent any kind of relation (dynamic, static, linear, or non-linear). (These relations are also called constraints as the value of an involved variable can not change independently of the other involved variables [3, 5]). The system's structural model is represented by the set of relations  $\mathcal{F} = \{f_1, f_2, \dots, f_m\}$  and the set of variables  $\mathcal{Z} = \mathcal{K} \cup \mathcal{X} = \{z_1, z_2, \dots, z_n\}$ .  $\mathcal{X}$  is the set of unknown variables and  $\mathcal{K} = \mathcal{U} \cup \mathcal{Y}$  is the set of known variables i.e. input/reference signals ( $\mathcal{U}$ ), and measured signals ( $\mathcal{Y}$ ).

The set of constraints  $\mathcal{F}$  is separated into  $\mathcal{F}_{\mathcal{K}}$ , those that apply *only* to known variables, and  $\mathcal{F}_{\mathcal{X}} = \mathcal{F} \setminus \mathcal{F}_{\mathcal{K}}$  that is the set of those constraints that include at least one unknown variable.

### 2.1 Structural model representation

The system's structural model can be represented by a bipartite graph,  $G(\mathcal{F}, \mathcal{Z}, \mathcal{A})$  where elements in the set of arcs  $\mathcal{A} \subset \mathcal{F} \times \mathcal{Z}$  are defined in a certain way. To specify the elements in the set  $\mathcal{A}$  in a useful manner, an additional property that is the calculability property, needs to be taken into considerations.

**Definition 1 Calculability property:** Let  $z_j, j = 1, \dots, p, \dots, n$  be variables that are related through a constraint  $f_i$ , e.g.  $f_i(z_1, \dots, z_n) = 0$ . The variable  $z_p$  is calculable if its value can be determined through the constraint  $f_i$  under the condition that the values of the other variables  $z_j, j = 1, \dots, n, j \neq p$  are known.

**Remark 1** This property, in general, has restrictive effects on representing components which contain terms such as sinusoidal and polynomials. One solution is to investigate the relations around the possible operating points of the system.

**Remark 2** In analytical mathematical analysis the definition for calculability property, def. 1, is quite similar to the solution for the following formulation: When can the equation  $f(x_1, \dots, x_n) = 0$  be solved explicitly for  $x_i$  in terms

of  $x_j$ ,  $j = 1, \dots, n$ ,  $j \neq i$ ? Conditions for the local solution is provided by the implicit function theorem ([1]). This theorem states that for a given function  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  and a local point  $\mathbf{x}_0$  (a possible operating point), where  $f(\mathbf{x}_0) = 0$  and for which  $(\partial f / \partial x_i|_{\mathbf{x}_0}) \neq 0$ ,  $1 \leq i \leq n$ , then there exists a function  $g$ , defined on  $\mathbb{R}$ , such that  $x_i = g(x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$ .

**Example 1** Consider the relation  $f(\dot{x}, u) = 0$  which represents the equation  $\dot{x} - u = 0$ . We would like to determine which of these two variables,  $x$  and  $u$ , can be explicitly calculated. Applying the implicit function theorem we get:

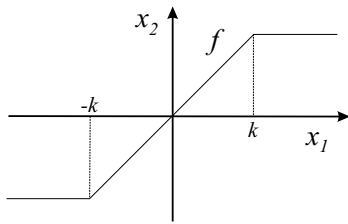
$$\frac{\partial f}{\partial x} = \frac{\partial(\dot{x} - u)}{\partial x} = \frac{\partial \frac{\partial x}{\partial t}}{\partial x} = \frac{\partial \frac{\partial x}{\partial t}}{\partial t} = 0$$

and

$$\frac{\partial f}{\partial u} = \frac{\partial(\dot{x} - u)}{\partial u} = -1 \neq 0$$

Obviously,  $u$  can be explicitly computed by knowing the values of  $x$  through the equation represented by the relation  $f$ . The opposite is not true:  $x$  can not be reconstructed explicitly using the values of  $u$  since  $x$  is given by  $x = \int u dt + x_0$  and the initial value  $x_0$  is not known (since  $x \in X$  which is the set of unknown variables).

**Example 2** The function  $f$  in Figure 2.1 represents a surjective mapping from  $x_1$  onto  $x_2$ . This mapping is not bijective. This means that the values for the variable  $x_2$  are always calculable for given values of  $x_1$ , but the inverse is not always possible.



b) Functional relation

Taking calculability properties into considerations, the systems structural model is now represented by a bipartite directed graph:

**Definition 2** The structure graph of the system is a bipartite directed graph  $(\mathcal{F}, \mathcal{Z}, \mathcal{A})$  where the elements in the set of  $\mathcal{A} \subset (\mathcal{F} \times \mathcal{Z})$ , where  $\mathcal{Z} = \mathcal{K} \cup X$  are defined by:

$$\begin{cases} a_{ij} = (f_i, x_j) = 1 & \text{iff } f_i \text{ applies to } x_j, \\ a_{ij}^* = (x_i, f_j) = 1 & \text{iff } x_i \text{ is calculable through } f_j \\ kf_i = (k_i, f_j) = 1 & \text{iff } f_j \text{ applies on a known var } k_i. \\ 0 & \text{Otherwise.} \end{cases}$$

for all  $x \in X$  and  $k \in \mathcal{K}$ .

The corresponding incidence matrix  $\mathbf{I}_{\text{md}}$  has the following compact form:

$$\begin{matrix} & \mathcal{K} & \mathcal{F}_X & X \\ \begin{matrix} \mathcal{K} \\ \mathcal{F}_X \\ X \end{matrix} & \begin{bmatrix} 0 & \mathbf{KF} & \mathbf{0} \\ \mathbf{KF}^T & \mathbf{0} & \mathbf{A} \\ \mathbf{0} & \mathbf{A}^* & \mathbf{0} \end{bmatrix} & = \mathbf{I}_{\text{md}} \end{matrix} \quad (2)$$

where  $\mathbf{A}$ ,  $\mathbf{A}^*$ , and  $\mathbf{KF}$  are given as:

$$\mathbf{A} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \quad (3)$$

$$\mathbf{A}^* = \begin{bmatrix} a_{11}^* & \dots & a_{1m}^* \\ \vdots & \ddots & \vdots \\ a_{n1}^* & \dots & a_{nm}^* \end{bmatrix} \quad (4)$$

$$\mathbf{KF} = [kf_1 \quad \dots \quad kf_m] \quad (5)$$

where  $m$  and  $n$  are the number of elements in  $\mathcal{F}_X$  and  $X$  correspondingly.

Let  $\mathcal{E}$  denote a set (such as  $\mathcal{F}_X$  or  $\mathcal{Z}$ ) and  $\mathbb{P}(\mathcal{E})$  denote the power set of  $\mathcal{E}$ . Then a subsystem  $(F, Q(F))$ ,  $F \in \mathbb{P}(\mathcal{F}_X)$  will be defined as

$$\begin{aligned} \mathbb{P}(\mathcal{F}_X) &\xrightarrow{\mathcal{Q}} \mathbb{P}(\mathcal{Z}) \\ F &\rightarrow Q(F) = \{z_j \mid \exists f_i \in F \text{ such that } (f_i, z_j) \in \mathcal{A}\}. \end{aligned} \quad (6)$$

### 3 The matching concept

The ultimate aim of representing the system in terms of structured graph is to obtain knowledge about the parts/subsystems with inherent redundant information that exists within the system. These parts can be analyzed in detail and the redundant information can then be manipulated for FDI and fault accommodation purposes.

Consider a graph  $G(F_X, X, A_X)$  representing a restricted part of the system's structured graph. Let  $a = (F_X(a), X(a))$  be the arc that connects a constraint  $F_X(a)$  with an unknown variable  $X(a)$ .

**Definition 3** The (sub)graph  $G(F_{X_m}, X_m, A_{X_m})$  is a matching on  $G(F_X, X, A_X)$ ,  $F_{X_m} \subseteq F_X$  and  $X_m \subseteq X$ , iff:

- 1)  $A_{X_m} \subseteq A_X$ ,
- 2)  $\forall a1, a2 \in A_{X_m} \mid a1 \neq a2 \Downarrow$   
 $F_{X_m}(a1) \neq F_{X_m}(a2) \wedge X_m(a1) \neq X_m(a2)$ .

A complete matching w.r.t.  $F_X$  is obtained when  $F_{X_m} = F_X$   
A complete matching w.r.t.  $X$  is obtained when  $X_m = X$ .

By applying matching one can decompose the system into three parts according to the following theorem:

**Theorem 1** [6] Any bipartite graph of finite external dimension can be uniquely decomposed:

- $G^+ = (F^+, X^+, A^+)$  such that  $Q(F^+) = X^+$  and a complete matching exists on  $X^+$  but not on  $F^+$ .
- $G^- = (F^-, X^-, A^-)$  such that  $Q(F^-) = X^- \cup X^+$  and a complete matching exists on  $X^-$  as well as on  $F^-$ .
- $G^- = (F^-, X^-, A^-)$  such that  $Q(F^-) = X^- \cup X^+ \cup X^+$  and a complete matching exists on  $F^-$  but not on  $X^-$ .

$G^+$  represents the part of the system with possible redundant information as  $|F^+| > |X^+|$ , where  $|F|$  denotes the cardinality of  $F$ . The unknown variables in  $X^+$  can be calculated in several ways by using the known variables. The subsystem(s) represented by  $G^+$  is said to be *over-determined*, as the number of relations exceeds the number of unknown variables. That means a variable  $x$  in  $X^+$  can be computed/calculated through different sets of relations (equations) in  $F^+$ , or seen from a graph-theoretical point of view, there are different paths from  $x$  to the known variables (see next section). This property can be used for FDI purposes: if a component, such as a sensor, fails the related variable can be computed/estimated via other sets of relations and be used in the control loop.  $G^-$  and  $G^-$  represent the parts with no redundant information. More discussion on this can be found in [5] and [2].

### 3.1 Matching procedure

There exists a number of algorithms to perform matching in the literature (see f.ex. chap. 6 in [4] and references herein). However, they can not be directly applied due to the problem of calculability property, def. 1. A dedicated matching algorithm is been developed to decompose the system into different parts according to theorem 1.

The main purpose of developing a matching algorithm is to identify the sub-graph  $G^+$  that represents the subsystem(s) which contain redundant information. The idea is depicted in figure 1 and the algorithm initiates the matching from the known variables. The figure illustrates the idea of making the unknown variable "known" by successively matching them to the known variables. First, variables  $x_1$  and  $x_2$  are matched to constraints  $f_1$  and  $f_2$  (full line). These variables become "known" as all the other variables that enter  $f_1$  and  $f_2$  are known. Hence, the new set of known variables can be considered as  $\mathcal{K}_{new} = \mathcal{K} \cup x_1 \cup x_2$ . Next,  $x_3$  and  $x_4$  are matched to  $f_3$  and  $f_4$  correspondingly (dotted line) and same argument can be used for further matchings (if needed). The matching procedure makes extensive use of the incidence matrix,  $\mathbf{I}_{md}$ , of the system's bipartite directed graph model. The algorithm repeats itself until one of the stop criteria, which are defined in theorem 1, are met.

### 3.2 Matching possibilities

Through extensive examination of the obtained results and the resulting incidence matrix, following observations are made and stated below:

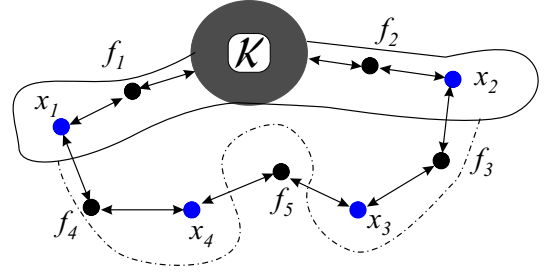


Figure 1: The process of matching.

Assume that a matching is performed on a system where a  $G^+ = (F^+, X^+, A^+)$  is identified. Suppose that  $|F^+| - |X^+| > 1$  then there exists at least two relations, let's say  $f_p$  and  $f_q \in F^+$ , that are not matched, otherwise there would be a contradiction with the matching definition Def. 3. Relations  $f_p$  and  $f_q$  represent two components (could be virtual components)  $C_p$  and  $C_q$  and each imply on a set of unknown variables, i.e.  $X_p$  and  $X_q$  where  $(X_p \cup X_q) \setminus X_p \neq \emptyset$ . By back-tracking each variable in  $X_p$  (corr.  $X_q$ ) to the known variables, via the matched pair of variables and relations, an over-determined subsystem,  $(F_p, Q(F_p))$  (corr.  $(F_q, Q(F_q))$ ) is identified. Since  $f_p \in F_p$  and  $f_q \in F_q$  and they are not the same (i.e. they represent different components) then  $F_p \neq F_q$ . Also since  $X_p \subset Q(F_p)$  and  $X_q \subset Q(F_q)$  and  $X_p \neq X_q$  then  $Q(F_p) \neq Q(F_q)$ . This proves the following statement:

**Proposition 1** For a system  $G^+ = (F^+, X^+, A^+)$  obtained by a matching:

1. The number of the over-determined subsystems obtained by back-tracking unknown variables, which are constrained by each unmatched relation in  $F^+$ , is:

$$|F^+| - |X^+|$$

2. and these subsystems are distinct.

It is important to mention that matching can generally be performed in several ways and different over-determined subsystems can be obtained by performing different matchings. However, the number of obtained over-determined subsystems is always the same for each performed matching, i.e.  $|F^+| - |X^+|$  as stated in prop. 1.

**3.2.1 Number of possible matchings:** Since different matching result in different over-determined subsystems, it is of interest to identify the number of matching possibilities. It is achieved by following the procedure below:

Denote the incidence matrix that corresponds to  $G^+$ ,  $\mathbf{I}_{md}^+$  with its affiliated matrices  $A^+$ ,  $A^{*+}$ , and  $KF^+$ . Then

- Identify all variables,  $x$ , that are measured directly, i.e. for each  $y_i \in \mathcal{Y}$ ,  $i = \{1, \dots, |\mathcal{Y}|\}$  there is a  $x \in X^+$  and a  $f \in F^+$  such that the relation  $f(y_i, x) = 0$  exists. Denote the set of all these  $x$ 's by  $X_m^+$ .
- Delete all the rows in  $A^{*+}$  that corresponds to the unknowns in  $X_m^+$ . Denote the resulting matrix  $A_{res}^{*+}$ .
- Perform Gauss-Jordan reduction to transform  $A_{res}^{*+}$  into its reduced row echelon form.
- The number of possible matchings,  $N$ , is calculated as:

$$N = \text{Nr. of non-zero elements in } A_{res}^{*+} - \text{columnrank}(A_{res}^{*+})$$

#### 4 Structural observability

Determining the value of a variable in the system is (structurally) possible if there exists a path from the unknown variable to a set of known (measured) variables, i.e.:

**Definition 4 (Structural observability):** A  $x_j \in X$ ,  $j \in \{1, \dots, |X|\}$  is structurally observable if and only if it is reachable from a set in  $\mathbb{P}(\mathcal{Y})$ .

Structural observability is in fact a generalization of calculability definition. An unknown variable can be structurally observed (and hence constructed) if it fulfills the following condition:

**Proposition 2** [7] *The necessary condition for a variable  $x \in X$  in the system  $S$  to be structurally observable is:*

$$x \in Q(F^+ \cup F^-).$$

Any over-determined subsystem is structurally observable due to the following lemma.

**Lemma 1** *Any over-determined subsystem that is obtained by back-tracking the unknown variables, which are constrained by any unmatched relation in  $F^+$ , is structurally observable.*

**Proof** Any state  $x$  in the over-determined subsystem belongs to  $X^+ = Q(F^+)$  and hence fulfills the necessary condition for observability (in accordance with prop. 2). The sufficiency is guaranteed by the way the matching procedure is performed; it starts with identifying the unknown variables which are measured and assumes them to be known and then repeats the loop until a stop criteria is met. ■

**Definition 5** *The minimal over-determined subsystem,  $G_{min-f}^+ = (F_{min-f}^+, X_{min-f}^+, A_{min-f}^+)$  is the smallest over-determined (hence observable) subsystem which is obtained by back-tracking the unknown variables in an unmatched relation  $f$  in  $F^+$ . For a minimal over-determined subsystem, the following statement is valid:*

$$|F_{min-f}^+| = |X_{min-f}^+| + 1.$$

#### 5 Fault detection and isolation

Any minimal over-determined subsystem yield an expression of following form

$$f(z_i, \dots, z_j) = 0 \quad z_i, \dots, z_j \in \mathcal{K} \quad (7)$$

where all involved variables are known. The expression can be directly used as an expression for a residual

$$r = f(z_i, \dots, z_j) \quad z_i, \dots, z_j \in \mathcal{K}. \quad (8)$$

The obtained residual can be directly used for detecting different faults.

Fault isolation possibilities can be examined by setting up a table that illustrates the effect of different faults on the set of residuals. The situation is exemplified in table 1. Results

	$\Delta z_1$	$\Delta z_2$	$\dots$	$\Delta z_{ K }$
$r_1$	1	1	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$r_N$	1	0	$\dots$	1

**Table 1:** Effect of different faults on residuals.

in obtained tables similar to table 1 can assist the designer to set up the required logic for fault isolation purposes.

#### 6 A linear system example

A Linear time invariant (LTI) dynamics system given by the following equations:

$$\dot{x}_1 = a_{11}x_1 + a_{13}x_3 + a_{14}x_4 + a_{16}x_6 \quad (9)$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{27}x_7 \quad (10)$$

$$\dot{x}_3 = a_{31}x_1 + a_{33}x_3 + a_{36}x_6 \quad (11)$$

$$\dot{x}_4 = x_2 \quad (12)$$

$$\dot{x}_5 = x_3 + a_{55}x_5 \quad (13)$$

$$\dot{x}_6 = a_{66}x_6 + b_{61}u_1 \quad (14)$$

$$\dot{x}_7 = a_{77}x_7 + b_{72}u_2 \quad (15)$$

$$y_1 = x_1 \quad (16)$$

$$y_2 = x_4 \quad (17)$$

$$y_3 = x_5 \quad (18)$$

with  $\mathbf{A}_s, \mathbf{B}$ , and  $\mathbf{C}$  being system matrices.  $\mathbf{y}$  and  $\mathbf{x}$  are the systems output and state vectors.

Each equation can be represented by a relation. For instance, Eq. 9 is represented by  $f_1(x_1, x_3, x_4, x_6) = 0$  and Eq. 18 is represented by  $f_{10}(x_5, y_3) = 0$ . The system's structural model is defined by using the following sets:

$$\begin{aligned}\mathcal{F} = \mathcal{F}_X &= \{f_1, \dots, f_{10}\} \\ \mathcal{X} &= \{x_1, \dots, x_7\} \\ \mathcal{K} &= \mathcal{U} \cup \mathcal{Y} = \{u_1, u_2\} \cup \{y_1, y_2, y_3\}\end{aligned}$$

**Figure 2:** Bipartite di-graph representation of LTI system’s structural model.

(20)

$$\begin{aligned}
A &= \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \\
A^* &= \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
KF^T &= \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \quad (21)
\end{aligned}$$

Following the procedure given in subsection 3.2.1, the number of matching possibilities according to Eq.7 is:

$A_{res}^{+*}$  is obtained by removing first, fourth, and fifth rows from  $A^*$ . Table 2 illustrates the matched pairs for each of the two possible matchings: The resulting matchings show

Match 1	match 2
$f_8 \longleftrightarrow x_1$	$f_8 \longleftrightarrow x_1$
$f_4 \longleftrightarrow x_2$	$f_4 \longleftrightarrow x_2$
$f_5 \longleftrightarrow x_3$	$f_5 \longleftrightarrow x_3$
$f_9 \longleftrightarrow x_4$	$f_9 \longleftrightarrow x_4$
$f_{10} \longleftrightarrow x_5$	$f_{10} \longleftrightarrow x_5$
$f_3 \longleftrightarrow x_6$	$f_1 \longleftrightarrow x_6$
$f_2 \longleftrightarrow x_7$	$f_2 \longleftrightarrow x_7$

**Table 2:** Matched pair for each possible matching for the LTI system

that  $X^+ = X$  and  $Q(F^+) = X^+ = X = Q(\mathcal{F}_X)$  and hence  $F^+ = \mathcal{F}_X$  for each matching. According to proposition 1, the number of distinct over-determined subsystems (for each performed matching) is given by  $|F^+| - |X^+|$  which is again equal to the number of unmatched relations. Table 3 shows the unmatched relations for each matching:

Matching Number	Unmatched relations
1	$f_1, f_6, f_7$
2	$f_3, f_6, f_7$

**Table 3:** Unmatched relations for each performed matching.

**6.2.1 Over-determined subsystems:** All over-determined subsystems can be identified as mentioned before via unmatched relations for all possible matchings. Referring to the results shown in table 3 one can at most obtain 6 over-determined subsystems. Tables 4 and 5 show all possible over-determined subsystems that are obtainable. Each row illustrates the involved relations (in table 4) and unknown variables (in table 5) in each case. The

	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$	$f_6$	$f_7$	$f_8$	$f_9$	$f_{10}$
1-1	1		1		1			1	1	1
1-6			1		1	1		1		1
1-7		1		1	1		1	1	1	1
2-3	1		1		1			1	1	1
2-6	1				1	1		1	1	1
2-7		1		1	1		1	1	1	1

**Table 4:** Possible over-determined subsystems - involved relations/equations .

first column in each table indicates matching and involved

unmatched relation. For instance, 2-6 indicates that the over-determined subsystem is obtained from unmatched relation  $f_6$  in matching number 2 (see table 3). There are

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$
1-1	1		1	1	1	1	
1-6	1		1		1	1	
1-7	1	1	1	1	1		1
2-3	1		1	1	1	1	
2-6	1		1	1	1	1	
2-7	1	1	1	1	1		1

**Table 5:** Possible over-determined subsystems - involved unknown variables.

two sets of rows that are identical in both tables, rows 1, and 4 and rows 3 and 6. Rows 4, and 6 can hence be disregarded.

### 6.3 Redundant relations and residual expressions

Each row in the remaining rows in the table represents a minimal over-determined subsystem, and contains a redundant relation that can be used for fault detection. When the redundant relation is analytical, as in the case of LTI system in this example, then analytical redundant relations (ARRs) can be directly deduced from the minimal over-determined subsystems in tables 4 and 5 in a sequential manner. To illustrate the idea, the ARR for the minimal over-determined subsystem in row (1-1), table 4, is deduced. Involved equations/relations are

$$\begin{aligned}
 f_1 : \quad & \dot{x}_1 = a_{11}x_1 + a_{13}x_3 + a_{14}x_4 + a_{16}x_6 \quad (22) \\
 f_3 : \quad & \dot{x}_3 = a_{31}x_1 + a_{33}x_3 + a_{36}x_6 \\
 f_5 : \quad & \dot{x}_5 = x_3 + a_{55}x_5 \\
 f_8 : \quad & y_1 = x_1 \\
 f_9 : \quad & y_2 = x_4 \\
 f_{10} : \quad & y_3 = x_5
 \end{aligned}$$

By replacing  $x_1$  with  $y_1$ ,  $x_4$  with  $y_2$ ,  $x_5$  with  $y_3$ , and getting rid of  $x_6$  from relations  $f_1$  and  $f_3$  following ARR is obtained:

$$\begin{aligned}
 \ddot{y}_3 + \underbrace{\left(\frac{a_{36}a_{13}}{a_{16}} - a_{33} - a_{55}\right)}_{\alpha_{y_3}} \dot{y}_3 + a_{55} \underbrace{\left(a_{33} - \frac{a_{36}a_{13}}{a_{16}}\right)}_{\beta_{y_3}} y_3 = \\
 - \underbrace{\frac{a_{36}a_{14}}{a_{16}}}_{\alpha_{y_2}} y_2 + \underbrace{\frac{a_{36}}{a_{16}}}_{\alpha_{y_1}} \dot{y}_1 + \underbrace{\left(a_{33} - \frac{a_{36}a_{11}}{a_{16}}\right)}_{\beta_{y_1}} y_1 \quad (23)
 \end{aligned}$$

A *residual* is the result of an ARR calculation when the known variables are replaced by their values. Residual expression for over-determined subsystem 1-1 can hence be written as

$$r_{1-1} = \ddot{y}_3 + \alpha_{y_3} \dot{y}_3 + \beta_{y_3} y_3 - \alpha_{y_2} \dot{y}_2 - \alpha_{y_1} \dot{y}_1 - \beta_{y_1} y_1 \quad (24)$$

Residual expression for the other over-determined subsystems in table 4 can be obtained in a similar manner.

**6.3.1 Residual evaluation:** Residual *evaluation* allows to explain the behavior of obtained residuals with respect to different faults. A practicable method is to directly inspect table 4. Fault in sensors and actuators will directly affect relations  $f_{10}$ ,  $f_9$ ,  $f_8$ ,  $f_7$ , and  $f_{10}$ . Table 6 illustrates possible effect of fault on different residuals. Here a “1” in an intersection signifies that the fault in the column affects residual in the corresponding row. It seems that faults  $\Delta y_1$

Residual	$\Delta u_1$	$\Delta u_2$	$\Delta y_1$	$\Delta y_2$	$\Delta y_3$
$r_{1-1}$			1	1	1
$r_{1-6}$	1		1		1
$r_{1-7}$		1	1	1	1
$r_{2-6}$	1		1	1	1

**Table 6:** Residual evaluation - Possible faults' effect on different residuals.

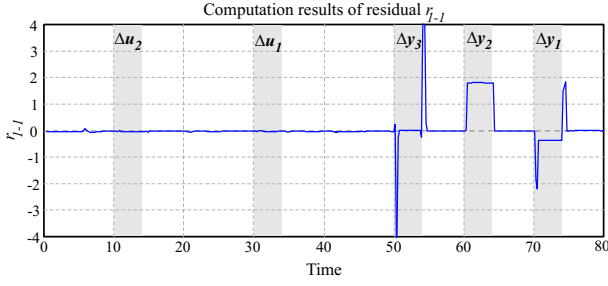
and  $\Delta y_3$  can not be isolated from each other. However, when analytical redundancy relations are available, an evaluation form can be used to obtain *sensitivity* expression for each residual by taking partial derivative w.r.t. “faulty” variables. The sensitivity expression can provide additional information that can be used for isolation purposes. The sensitivity expression for residual  $r_{1-1}$  in Eq. 24 is

$$\begin{aligned}
 r_{1-1,s} = & \frac{d^2 \Delta y_3}{dt^2} + \alpha_{y_3} \frac{d \Delta y_3}{dt} + \beta_{y_3} \Delta y_3 \\
 & - \alpha_{y_2} \frac{d \Delta y_2}{dt} - \alpha_{y_1} \frac{d \Delta y_1}{dt} - \beta_{y_1} \Delta y_1
 \end{aligned}$$

An abrupt or incipient sensor fault in sensors 1 and 3 will have an immediate impact on the residual; the mean value of the residual will change immediately under the condition that  $\beta_{y_3}$  and  $\beta_{y_1}$  are not zero. On the other hand an abrupt fault on sensor 2 (with step-like appearance) will only generate an impulse-like change in the residual that is quite hard to detect in a robust manner. An incipient fault with slow dynamic on sensor 2 will not have an impact on this residual and hence can not be detected.

## 7 Simulation results

Equations 9–18 represent the linear model of motion of a fixed-wing aircraft dynamics given in [8]. The states are,  $x_1$ : sideslip velocity,  $x_2$ : roll rate,  $x_3$ : yaw rate,  $x_4$ : roll angle,  $x_5$ : yaw angle,  $x_6$ : rudder angle,  $x_7$ : aileron angle. The control commands are,  $u_1$ : rudder angle command, and  $u_2$ : aileron angle command. The non-zero elements in  $\mathbf{A}_s$ ,  $\mathbf{B}$  are,  $a_{11} = -0.277$ ,  $a_{13} = -32.9$ ,  $a_{14} = 9.81$ ,  $a_{16} = -5.432$ ,  $a_{21} = -0.103$ ,  $a_{22} = -8.325$ ,  $a_{23} = 3.750$ ,  $a_{27} = -28.640$ ,  $a_{31} = 0.365$ ,  $a_{33} = -0.639$ ,  $a_{36} = -9.490$ ,  $a_{42} = 1$ ,  $a_{53} = 1$ ,  $a_{55} = 0$ ,  $a_{66} = -10$ ,  $a_{77} = -5$ ,  $b_{61} = 20$ , and  $b_{72} = 10$ .



**Figure 3:** Calculated values of residual  $r_{1-1}$

It should be noticed that the values of the parameters does not affect the results of the performed analysis in the previous section. They only affect the computation results of the residuals.

Values of the residuals in table 6 can be obtained by direct computation. As an example, the residual  $r_{1-1}$  is computed and illustrated in figure 3. All, on the figure, indicated faults are additive steps and are generated in periods of 4 seconds. Their values are:  $\Delta y_1 = 4$ ,  $\Delta y_2 = 1$ ,  $\Delta y_3 = 0.4$ ,  $\Delta u_1 = 0.2$ , and  $\Delta u_2 = 0.1$ . Measurement noise is not simulated to enhance visibility. Figure 3 illustrates the expected effect of sensor faults on the residual. Dynamic transient effect due to fast change in set points can be handled by choosing an appropriate threshold.

## 8 Conclusions

The paper has presented a novel approach to fault diagnosis. The goal was to obtain systems' inherent redundant information that can be used for FDI purposes. This was achieved by establishing the system's structural model where the system was represented by a bipartite directed graph. The matching concept was utilized in a specific manner to identify the structurally observable part of the system. Any unmatched relation in the obtained observable system can be used to generate a redundant information, which can further be manipulated to obtain a residual. A LTI model of motion of a fixed-wing aircraft was used to illustrate the steps in this approach. It was possible to obtain expressions for 4 different residuals. Simulation results for one of the residuals, using the numerical values, were given.

The presented approach provides a powerful tool for analyzing the systems at any stage of the design as it does not depend on detailed information. Also, as this approach provides the result by manipulating the defined incidence matrix, and the fact that the involved elements are either 0 or 1, the complexity and size of the system will not pose any difficulties. Disturbances and fault estimation issues are the topic of future work.

## Notation

$C_i$	$i^{\text{th}}$ system component
$f_i$	relation governed by $C_i$
$\mathcal{F}, \mathcal{X}, \mathcal{K}, \mathcal{U}, \mathcal{Y}$	sets of: relations, unknown variables, known variables, inputs, measurements
$G(F, X, A)$	bipartite directed graph
$g(\cdot)$	function (not relation)
$\mathbb{P}(\mathcal{E})$	power set of $\mathcal{E}$
$(F, Q(F))$	a subsystem in the system $(\mathcal{F}_X, \mathcal{X})$
$\mathbf{I}_{\text{md}}$	incidence matrix of $G$
$\mathbf{A}, \mathbf{A}^*, \mathbf{K}\mathbf{F}$	sub-matrices in $\mathbf{I}_{\text{md}}$
$N$	nr. of possible matchings
$ \mathcal{E} $	nr. of elements in set $\mathcal{E}$
<b>Subscripts</b>	<b>indicates or relates to</b>
$i, j, p, n$	function or variable indices
$m$	indicates a relation to a matching
<b>Superscripts</b>	<b>indicates or relates to</b>
+	over-determined subsystem
=	just-determined subsystem
-	under-determined subsystem

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